Basics of Astrophysics

Chapters 1-3

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Chapter 1

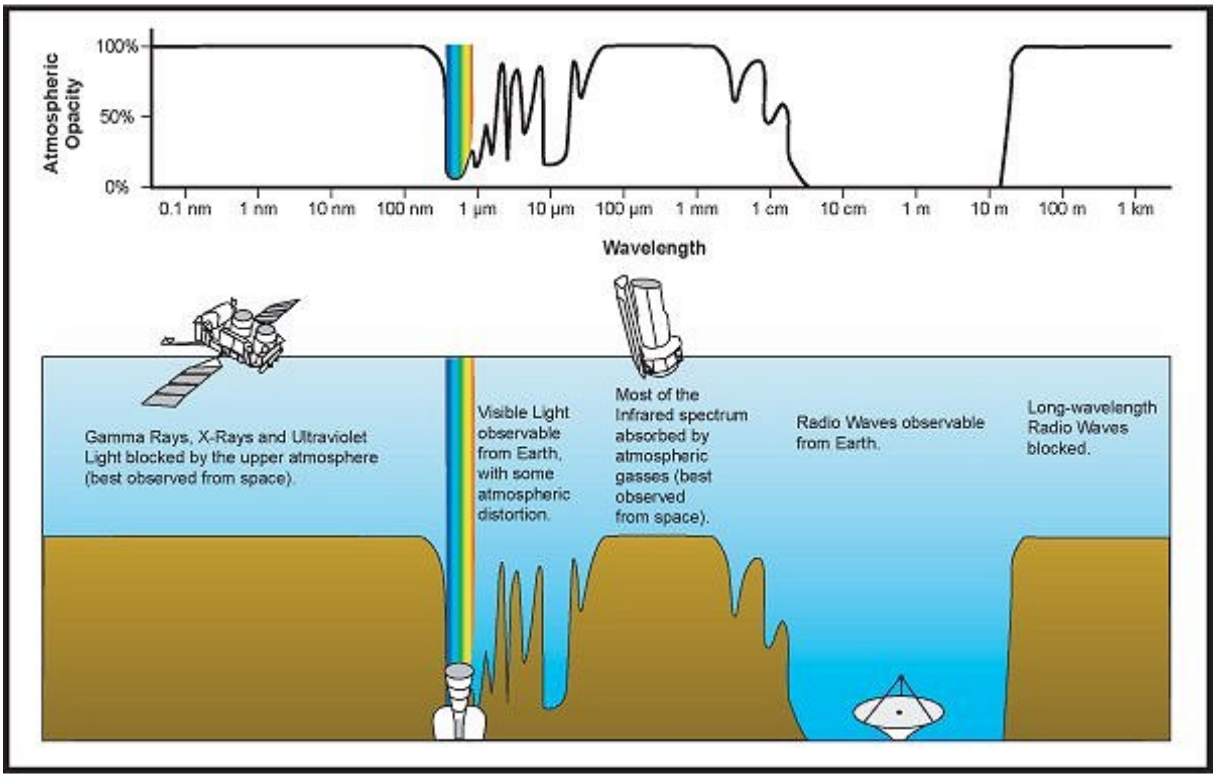
Sources of information in astronomy,

Observation and radiation essentials

A. Observational Technics

1. The electromagnetic spectrum

Astronomical phenomena are almost studied through their electromagnetic emission.
Saying that exclude the observation of cosmic rays or neutrinos and the tentative detection of gravitational waves.
Photons at different wavelengths are collected with telescopes and measured with a camera.
The instruments are different according to the wavelength range which is studied (X-ray, optical, infrared etc…).
The astronomical observation is dependent on the transmission of the Earth's atmosphere.



This figure shows the opacity of the atmosphere as a function of the wavelength. The visible light corresponds to a very limited domain of the electromagnetic spectrum. Its privileged role is due to the high transmission of the atmosphere (but not as high as for radio waves) and the performances of the human eye. Galileo refracting telescope used the eye as detector. Some parts of the spectrum are totally opaque, all the wavelengths shorter than the visible one (UV to γ rays) and from 10 µm to few mms. Radio waves can reach the earth without any absorption. Radio wavelength are large in comparison to the typical size of atmospheric particles and atmospheric effects are negligible. For wavelength larger than 10 m the ionosphere reflects the waves.
When one wants to observe EM radiation out of the transparent windows of the atmosphere, he has to send his telescope above it, on board of a satellite.

2. Measuring the signal: telescope characteristics

a. Telescope size

Telescope become bigger and bigger. There are two main reasons to this increase: the first one is the collection of photons, astronomical sources are distant and faint objects and the size of the collecting area is important. The aperture diameter of a human pupil (at night) is < 1 cm, and the largest telescopes today have a 8-10 m diameter mirror.

L is the luminosity of an object (in W), for example the luminosity of the sun is

$$L_{sun} = 3.846\ 10^{26}\ W$$

The flux f of a source is the radiative energy per unit time passing through unit area:

$$dL = f\ dA, \quad f \text{ is expressed in } W\ m^{-2}$$

This relation holds for bolometric quantities. For luminosity of flux we can define quantities in a given waveband or per unit wavelength or frequency:

$dL = L_\nu\ d\nu$ or $dL = L_\lambda\ d\lambda$, $df = f_\nu\ d\nu$ or $df = f_\lambda\ d\lambda$,

One can also write $dL_\nu = f_\nu\ dA$ (or with λ)

A special unit of flux density is called the *Jansky (Jy)* :

1 Jy corresponds to $10^{-26}\ W\ m^{-2}\ Hz^{-1}$

The wavelength is commonly measured in Angstroms or nms for the optical light, μm or mm for IR-submm waves

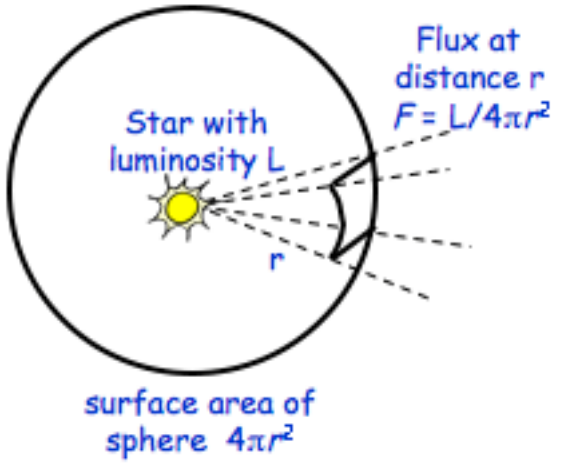
One can write $L = \int f\ dA$

The assumption of an isotropic emission is verified for a star (most of the time), it may not be the case for other objects, but **it is usually assumed to be true.**

If the source is isotropic (photons escaping in all directions) then

$$\mathbf{L = 4\ \pi\ r^2\ f}$$

Let us also assume that the distance between the object and the telescope is r and that the radius of the telescope is R

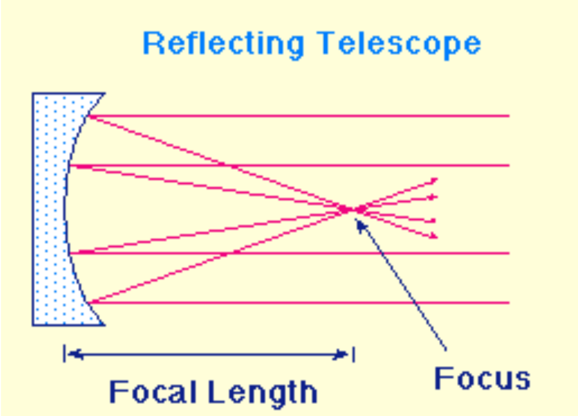


Then the collected luminosity on the telescope is $L_T = f * 4\ \pi\ R^2 = L * 4\ \pi\ R^2/4\ \pi\ r^2$

So the intensity of the signal increases with the square of the radius (or diameter) of the telescope and more distant sources are reachable.

b. <u>Very brief description of a telescope</u>

The telescopes used for astronomy are reflecting systems. The astronomical sources being distant the rays are parallel, the concave mirror focuses the light in its focal plane where the detector is put. Note that other (more complex) configurations exist.



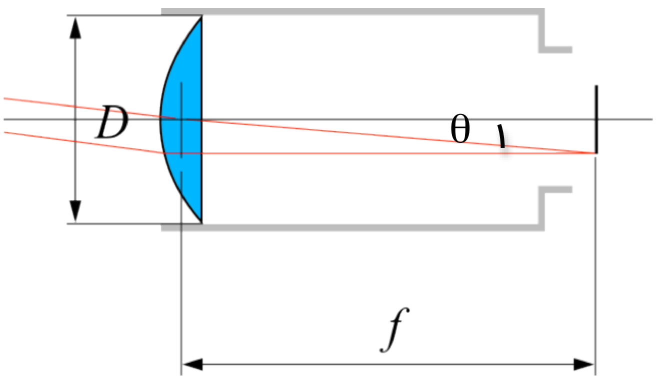
A telescope are designated by their diameters but also by their focal ratios or f-number N= f/D

For example a 3 m telescope with N = f/5 has a focal length equal to 15 m.

The focal length is important since is linked to the linear size of an object on the detector.

For clarity we have considered a refracting telescope (simple lens): the angular size of the object on the sky is θ, and the linear size of the image on the detector in the focal plane is l = θ* f (θ is a small angle assumed to be equal to tan θ).

The size of the detector is designed according to the characteristics of the telescope and the size of the field that one wants to observe.



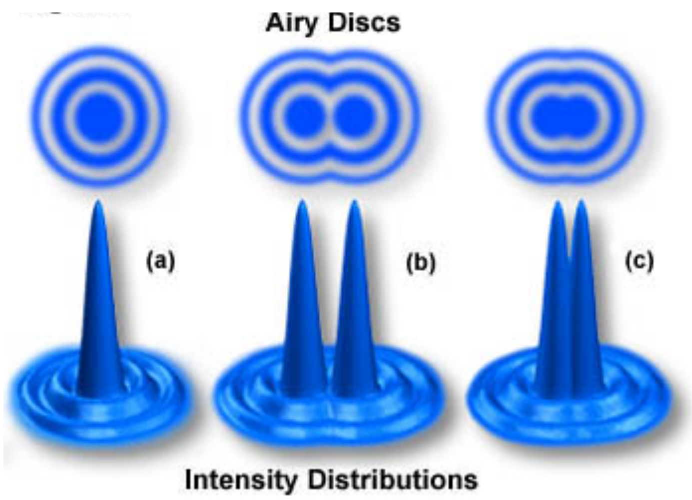
Angular resolution

The angular resolution of a camera is the smallest angle on the sky between two sources of light which can be detected as separate sources.

The diffraction pattern created by a circular aperture is Airy pattern: the angular distance from the first null to the central point is:

$\theta = 1.22\ \lambda/D$ and the full width half maximum of the central peak is $\theta_{FWHM} = 1.02\ \theta/D$

The central peak is called the Airy disk, for a uniform aperture illumination the Airy disk contains 84% of the total energy falling on the detector.



If the Airy disks of two adjacent sources overlap they will not be seen as separate sources. The minimum angle on the sky at which it is still possible to distinguish the sources is called the diffraction limit, it is an ideal case since the image quality is usually worse than the "ideal" one only limited by the size of the optics.
The diffraction limit is often specified with **the Rayleigh criterion** which states that the two spots can be distinguished when the maximum in one Airy pattern corresponds to the first minimum of the second one:

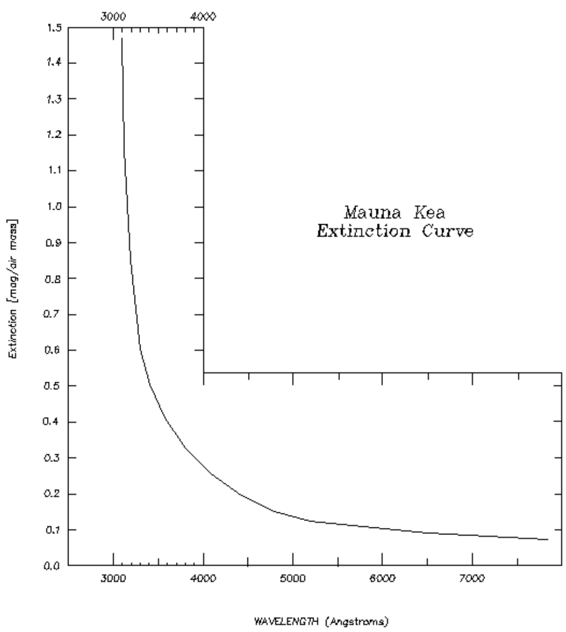
$$\theta_{min} = 1.22\, \lambda/D \approx \lambda/D$$

3. The atmosphere

Any observation from the earth must deal with the effects of the atmosphere. These effects are due to the interaction of the photons with the matter inside atmosphere, and are wavelength dependent, which greatly affects the signal received. The spectral response (tranmission function) of the atmosphere is shown above (section A.1), only radio and optical observations are conducted from the ground. Radio wavelengths are very large (>10 cm) as compared to the size of molecules in the atmosphere, and the atmosphere is transparent for these waves. The optical window (near UV to near-IR) is strongly affected by the transmission through the atmosphere, and telescopes are usually put on top of high mountains to limit its impact

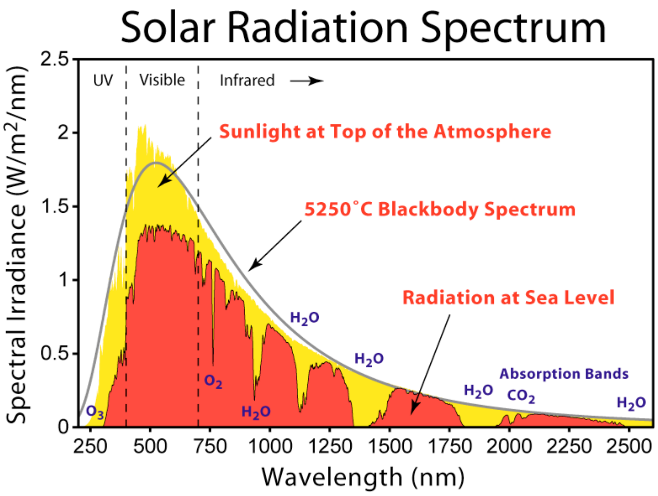
a. <u>Extinction, air mass</u>

Light travels through the atmosphere an molecules interact with it through absorption and scattering. For example ozone strongly affects the UV light. The main contributor is the water vapour (with a smaller contribution of the other molecules present in the atmosphere) molecular nitrogen, oxygen and carbon dioxide add to this process. By the time it reaches the Earth's surface, the spectrum is strongly confined between the far infrared and near ultraviolet.



The extinction curve of one of the best sites in the world is shown besides: the extinction strongly increases from the near IR to the near-UV.
The effect increases with the path length through the atmosphere.
For a path length L the airmass (AM) is defined as the ratio of the actual path length to the shortest one at the same location, L_0, i.e. the one normal to the earth's surface :

$AM = L/L_0 = 1/\cos z$, z being the zenith angle
The extinction curve is given per air mass unit



This figure illustrates the effect of the atmosphere on the sunlight. In yellow is the emission of the sun and the curve is the black body model, the sunlight at the sea level is shown in red, the main absorption bands are due to H_20.

b. <u>Seeing</u>

The atmosphere is turbulent and can be schematized as cells of gas in constant motion. Above the systematic refraction there is a random one. The EM wave is distorted producing a distorted image. If there are several cells in front of the aperture, several distorted images are formed and overlap. Each individual distorted image is called a speckle. A typical size for a cell is 10 cm and can go up to 20 cm at high altitude, where the atmosphere is more stable.
The structure of the speckle can be easily understood by the diffraction modelling:
Let us consider a large number of identical cells randomly distributed. The planar wave coming from the star is diffracted by this transmission screen, and the resulting intensity distribution in the focal plane is a random distribution of individual images whose size is determined by the diffraction pattern caused by the optical system (the telescope). The global distribution of the light in the complete image (which consists of all the individual images) corresponds to the size of each grain, just like in the case of a multi slit grating.

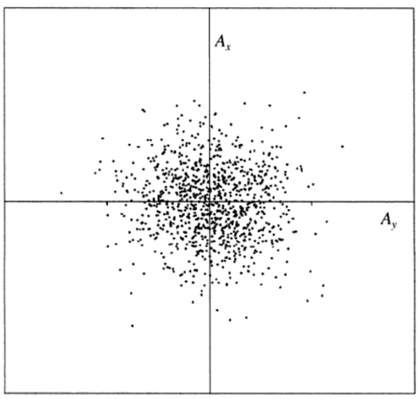
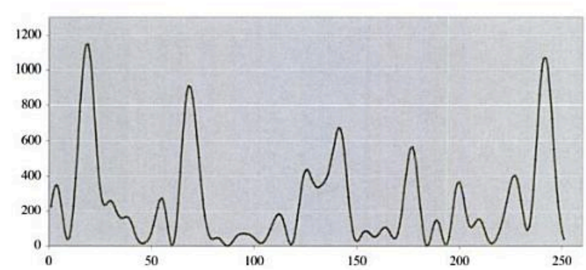


FIG. 7.8 **Variation de l'intensité lumineuse le long d'une droite dans une figure de speckle.**

Cette courbe a été tracée à partir d'une série de nombres aléatoires obéissant à une distribution normale.



The speckle pattern change rapidly, if v is the speed of the wind the timescale can be roughly estimated by $t = r/v$ where r is the size of the cell causing the speckle.
With a rather stable atmosphere ($v= 10$ km h^{-1}) and a cell size of 10 cm $t= 0.04$ s.
Exposure times are much larger than this value, during an exposure time of several seconds or more the speckle are shifting producing a smeared out image. The characteristic size of this pattern is taken as the seeing disk and assumed to be the FWHM of the seeing to be compared to other image sizes.
Telescopes on the ground are limited by the seeing and not by the diffraction.

The seeing θ_s can be roughly estimated:

For a cell of size r the size of the Airy disk is approximately λ/r giving at 0.5 μm and for r=10 cm it gives a typical value of 1 arcsec.

So any telescope with a diameter larger than 10 cm and working in visible will be seeing limited from the ground. However diffraction limited images are routinely obtained in infrared and radio astronomy where the effect of the atmosphere is strongly reduced.

c. Adaptative optics

To avoid this image deterioration due to the seeing, one solution is to launch telescope out of the atmosphere. It was the case for example for the Hubble Space Telescope which reached diffraction limit.
Another method is called adaptative optics. It is based on a correction of the seeing on real time and is mostly used in the near-IR. A deformable mirror is shaped in order to compensate the wave front distortion. The correction is performed by using a reference signal which must be close enough to the astronomical source to pass through the same atmosphere. It is not easy to find such bright stars and laser guide stars are used.
The image quality achieved with adaptative optics is very good and such a correction is now implemented on all the largest telescopes.

B. Panchromatic observations:

1. Photometric observations

a. Units

We have defined the flux of an astronomical source, the units used to define the value of this flux are multiple.
The flux itself can be given in W m^{-2} , you can still find it in cgs (erg $cm^{-2}s^{-1}$).
Per unit of frequency or wavelength, the Jansky (Jy) is now commonly used.

1 erg $s^{-1} = 10^{-7}$ W 1 erg $cm^{-2}s^{-1} = 10^{-3}$ Wm^{-2} 1 Jy = 10^{-26} W m^{-2} Hz^{-1} = 10^{-23} erg cm^{-2} s^{-1} Hz^{-1}

b. Definition of the magnitudes

Traditionally magnitudes are used to characterize light in the optical regime (from the UV to the near-IR). The origin of this logarithmic system is the human eye's response.
It is still widely spread out and must be well understood by anyone wanting to study astronomy.

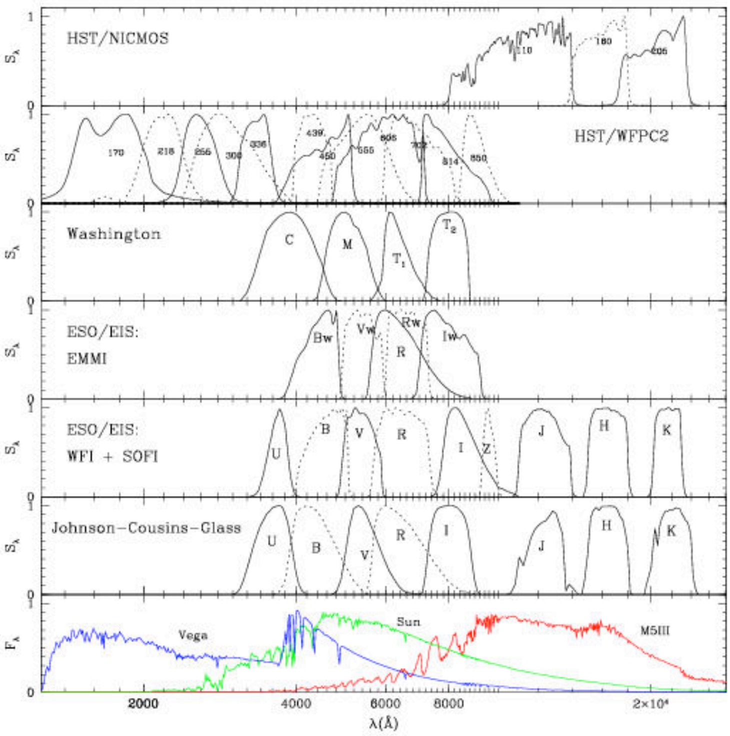
A flux is compared to a reference (calibrator).

$$m_\lambda - m_{\lambda_0} = -2.5 \times \log\left(\frac{f_\lambda}{f_{\lambda_0}}\right)$$

or $$m_\nu - m_{\nu_0} = -2.5 \times \log\left(\frac{f_\nu}{f_{\nu_0}}\right)$$

The subscript 0 holds for the calibrator, m are the magnitudes and f_λ and f_ν the flux densities (per wavelength or per frequency).
This system is relative, not absolute. The calibration depends on the filters, and some systems are pre-defined like the U,B,V,R,I,J,H,K,L system (Cousin-Glass-Johnson system).
However, other systems exist sometimes only slightly different from this one.
Practically each telescope has its own filter with a specific bandpass response which is calibrated and provided to the user.
Hereafter are shown several sets of filters, including the Johnson-Glass-Cousin system

Girardi et al. Astronomy & Astrophysics, 2002, 391, 195 Spectral response of different sets of filters, the lowest panel represents the spectrum of 3 stars: Vega, the sun and a cool M star.



The filters are defined by a response (or transmission) function S(λ) or S(ν).

The flux density (per unit wavelength) measured in a filter is defined as

$$f_\lambda = \frac{\int_0^\infty f(\lambda)\, S(\lambda)\, d\lambda}{\int_0^\infty S(\lambda)\, d\lambda}$$

f_λ is the flux density usually defined at the effective wavelength of the filter λ_{eff}

$$\lambda_{eff} = \frac{\int_0^\infty \lambda\, f(\lambda)\, S(\lambda)\, d\lambda}{\int_0^\infty f(\lambda)\, S(\lambda)\, d\lambda}$$

Note that the effective wavelength depends on the spectral distribution of the source

We need to define a reference to calculate the magnitudes:
The standard calibrator is chosen to have a magnitude equal to 0 in all wavebands.
Historically the reference was chosen to be Vega (αLyr) or the spectrum of an A0 star.

An example of reference flux densities is given in the table below, for an A0 star.
The magnitudes are expressed as a function of the zero-point magnitudes calculated in the corresponding filters:

Table A2. Effective wavelengths (for an A0 star), absolute fluxes (corresponding to zero magnitude) and zeropoint magnitudes for the UBVRI-JHKL Cousins-Glass-Johnson system

	U	B	V	R	I	J	H	K	Kp	L	L*
λ_{eff}	0.366	0.438	0.545	0.641	0.798	1.22	1.63	2.19	2.12	3.45	3.80
f_ν	1.790	4.063	3.636	3.064	2.416	1.589	1.021	0.640	0.676	0.285	0.238
f_λ	417.5	632	363.1	217.7	112.6	31.47	11.38	3.961	4.479	0.708	0.489
$zp(f_\lambda)$	0.770	-0.120	0.000	0.186	0.444	0.899	1.379	1.886	1.826	2.765	2.961
$zp(f_\nu)$	-0.152	-0.602	0.000	0.555	1.271	2.655	3.760	4.906	4.780	6.775	7.177

From Bessel, Castelli & Plez, Astronomy and Astrophysics, 1998, 331, 231

$m_\lambda = -2.5 \log(f_\lambda) - 21.100 - zp(\lambda)$

$m_\nu = -2.5 \log(f_\nu) - 48.598 - zp(\nu)$

f_λ (erg cm^{-2} s^{-1}A^{-1}) and f_ν (erg cm^{-2} s^{-1}Hz^{-1})

At UV and IR wavelength the definition of standard stars is more difficult. A more convenient, “flux-based” system has been developed to get rid of these calibrations.
It is called “AB system”, it corresponds to all the zero points set at 0.
In this new system only the V magnitude corresponds to the classical, old system (called Vega system).
The main advantage of this new system is to give direct access to physical (flux densities) quantities

AB system:

$m_\lambda = -2.5 \log(f_\lambda) - 21.100$

$m_\nu = -2.5 \log(f_\nu) - 48.598$

f_λ (erg cm^{-2} s^{-1}A^{-1}) and f_ν (erg cm^{-2} s^{-1}Hz^{-1})

Note on the units: unfortunately the international system is not used with magnitudes and fluxes are given in erg cm^{-2} s^{-1} instead of Wm^{-2}. *Sometimes the fluxes are given in Jy, mJy or μJy. In these lectures I use the international system each time it is possible,*

c. Absolute magnitudes and colour index

The previous definition of the magnitude, called 'apparent magnitude' does not involve the distance of the source. An *absolute magnitude* has been introduced, either as a bolometric quantity M_{bol} (i.e. integrated over all the wavelengths,) or in a given bandpass (M_B, M_V etc…). The absolute magnitude of a source is the magnitude that would be measured if this source is located at a distance of 10 pc.
Most of the time one defines the distance modulus as the different between the absolute and apparent magnitude:

$m\text{-}M = -2.5 \log(f/f_{10pc}) = -2.5 \log(10^2/d_{pc}^2) = -5+5\log(d_{pc})$
where d is expressed in pc

The colour index of a source is defined as the difference between two magnitudes (apparent or absolute since it does not depend on the distance of the source) measured in different filters.
For example $B\text{-}V = -2.5 \log(f_B/f_V)$ in the AB system
Or $B\text{-}V = -2.5 \log(f_B/f_V)-zp(B)+zp(V)$

The color index can be generalized by comparing the bolometric magnitude (over all the wavelength) to the one observed in one single bandpass. It is called the *bolometric correction* BC
For example in the V band: $BC = m_{bol}-V = M_{bol}-M_V$

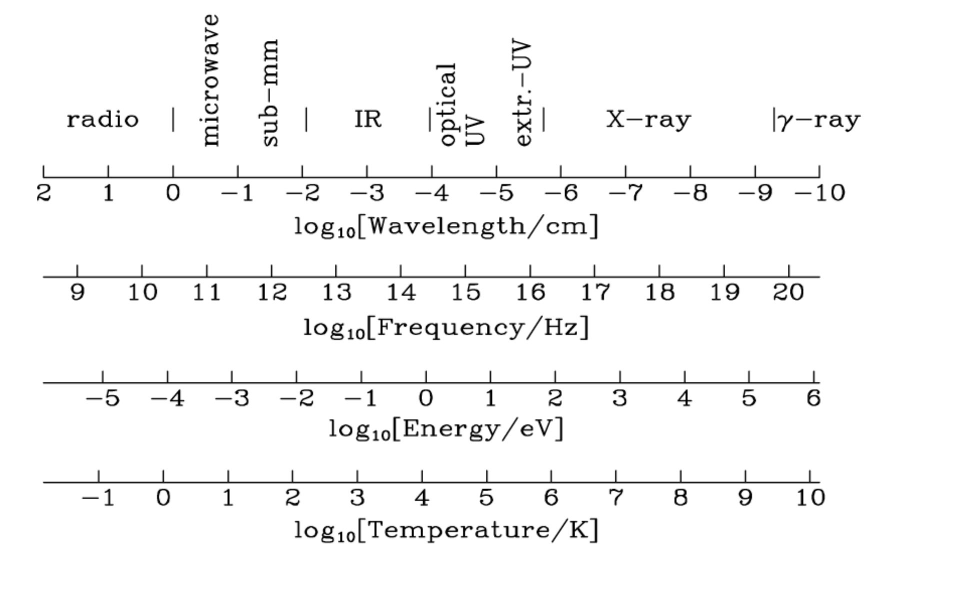
2. Spectroscopic observations

Astronomical spectroscopy is of fundamental importance. The observation of many emission and absorption lines give information on the physical content and conditions of the medium. Moreover the shift of the lines with the distance of the source (see next chapter) is the only way to measure accurately the distance of extragalactic sources.

C. Radiation essentials:

1. Wavelength coverage

Astronomical sources are radiating from the radio to the γ-rays covering 12 decades in wavelengths.
Units specific to the spectral range of interest are defined: wavelengths go from the nm in UV, µm in optical. eV are commonly used in high energy observations (keV, MeV, GeV) and temperature in Kelvin in IR to radio wavelengths.



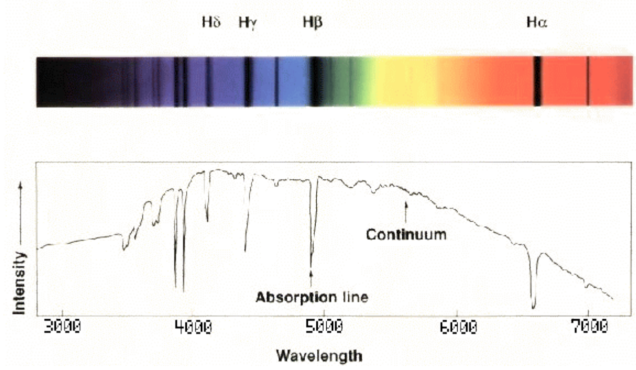
The various spectral regions of electromagnetic radiation, their common astronomical nomenclature, and their approximate borders in terms of wavelength, frequency, energy, and temperature. Temperature is here associated with photon energy E via the relation $E = kT$, where k is Boltzmann's constant.

2. A brief overview of the emission processes

The astronomical sources span a large range of physical quantities in terms of temperature (as an example several millions of degrees inside stars, ~10 K in cold gas clouds) or density (in the interstellar medium the density of molecular clouds is of the order of 10^{10} m^{-3} to ~10^{5} m^{-3} in the inter-cloud medium, the typical central density of stars is of the 10^{30} m^{-3}). As a consequence very different radiation processes are at work which depend on the medium. Here is a very brief panorama of these different kinds of emission, we will go back in detail to some of them, some processes are also described in other courses.

<u>a. Dense medium</u>

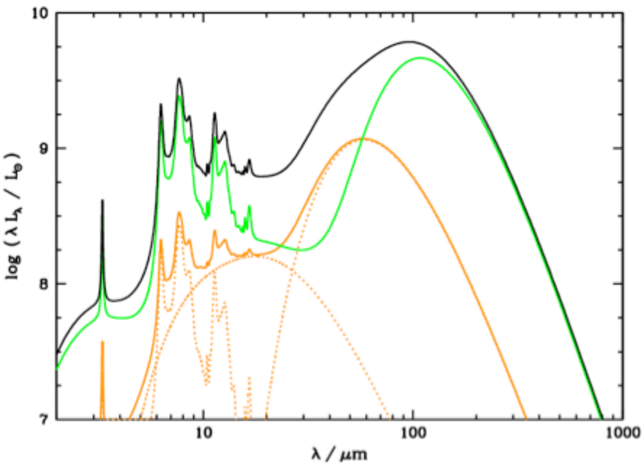
A dense medium, like the one of stars is essentially emitting like a black body (note that it is the emission of the surface of the stars, not the central emission). The effective temperature of stars (the temperature defined from the black body emission) can go from 20 000 K to few thousands of Kelvin. Applying the Wien law, the peak of the emission goes from the UV to the NIR.



Emission of a "typical" star with a continuum emission well described by a black body emission and absorption lines overimposed. Note the presence of absorption lines on the continuum.

Star birth occurs in dense media, specifically molecular clouds, and small particles, called dust grains, populate these cocoons. Such particles are also spread out in the medium between stars (the interstellar medium), they are heated with photons emitted by stars and re-emit the energy as modified black bodies. They are much colder than the stars (10-50 K) and radiate in the mid and far IR.
These emissions are essentially continuous as illustrated below (based on black body emission)

from http://www.sedfitting.org.
Model of the emission of the dust component, above ~15 μm the distribution is well described by a (modified) black body, below 15 μm the complex spectrum is due to Polycyclic Aromatic Hydrocarbons (PAHs) which are very large molecules with strong emission features in the mid-IR.



http://casswww.ucsd.edu/index.php/Main_Page

b. Diffuse medium

The interstellar medium is essentially diffuse.
The interaction of emitted photons with matter lead to various processes and different types of emission.
The resulting emission is either a continuum emission more or less featureless (as the ones described above) or line emission at specific wavelengths.

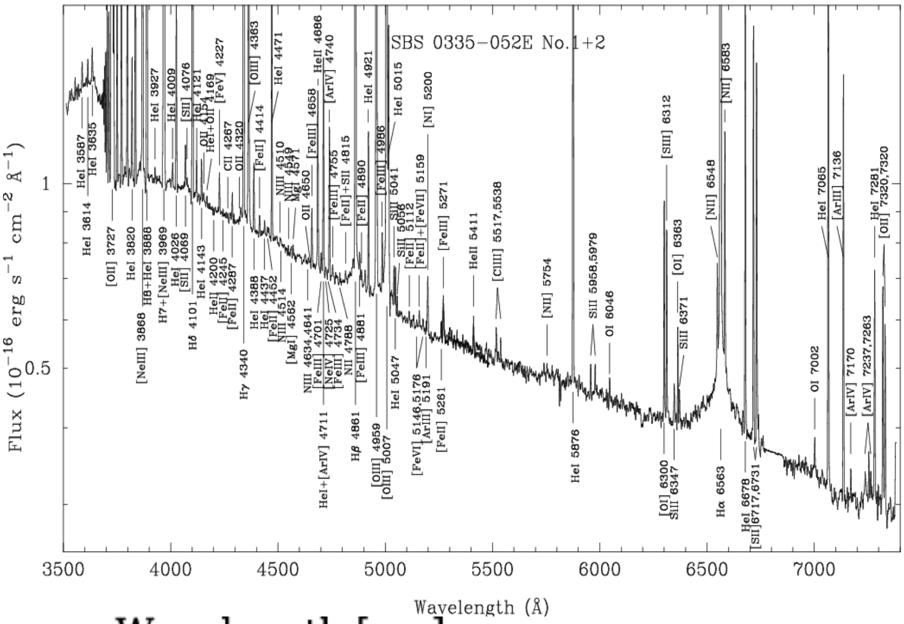
***Continuum emission is caused by an acceleration of a charged particle which is not bound**. There are two main categories of emissions: thermal emission and non thermal emission.
'Thermal emission' is given to any process from a medium described with a Maxwell-Bolztamm distribution. A (kinetic) temperature is then defined and the emission depends on this temperature. The black body emission is an example of thermal emission
Non thermal emission is related to all other processes, either because a temperature cannot be defined (velocity distributions which are not Maxwellian) or because the emission depends on an electric or magnetic fields.
The aim of this chapter is not to give a detailed description of all these processes but only to list the most important ones:
-free-free (Bremsstrahlung) emission: a free electron is accelerated in the vicinity of a charged nucleus, it is a thermal emission.
-free-bound (recombination) emission: capture of a free electron by a nucleus
-Synchrotron radiation: for relativistic electrons moving in a magnetic field, non thermal emission.
-Inverse Compton emission results of the interaction of a particle (high energy electron) with a (low energy) photon, it is a non thermal process

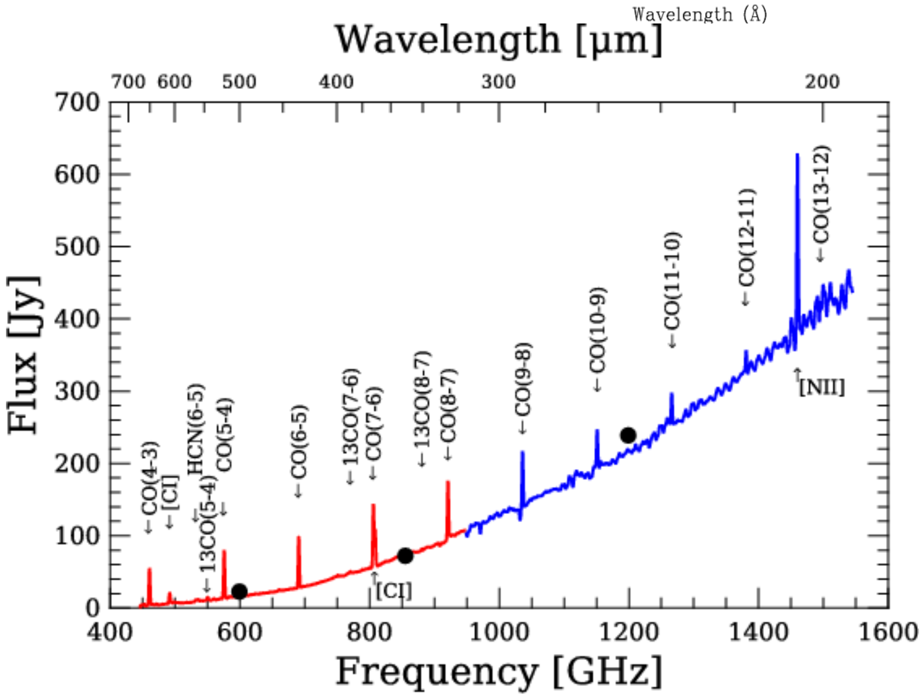
***Line emission comes from bound-bound transitions**

The lines are found from radio-waves to X rays and often over-imposed on the continuum

- Atomic electronic transitions: mostly in UV and visible and NIR, they occur in atoms and in molecules. Hydrogen lines are of fundamental importance but other atoms and molecules are also identified by their transition lines, considering all the species cover the entire electromagnetic spectrum.
- Molecular, rotational and vibrational line transitions: IR and mm emission lines, more than 100 molecules are observed in the interstellar medium,
- Nuclear transitions: high energy, γ-ray emission

Example of a spectrum of a galaxy with numerous electronic emission lines.





Rotational transition lines of the CO molecules in the far-IR from Panuzzo et al. 2010 A&A , 518, L37

Hereafter, a summary of the different sources of light in galaxies from Boselli, a panchromatic view of galaxies, Wiley

Table 1.1 Emitting sources and emission processes at different wavelengths.

Domain λ	X-ray 0.1–10 keV (1–100 Å)	UV 912–3500 Å	Visible 3500–7500 Å	NIR 0.75–5 µm	MIR 5-20 µm	FIR-submm 20 µm–1 mm	Radio 1 mm–1 m
				Continuum			
Process	Black body	Black body	Black body	Black body	Thermal emission	Modified black body	Synchrotron
	Thermal bremsstrahlung						Free–free
Emitting source	Accretion disk in binary systems	Young stars	Intermediate age stars	Old stars	PAH, hot dust grains	Cold dust grains	Relativistic electrons in weak magnetic fields
	Hot gas						HII regions
				Main emission lines			
Emission lines		Atomic hydrogen, metals	Atomic hydrogen, metals	Atomic and molecular hydrogen, molecules	PAH	[CII], CO, molecules	HI(21 cm)
Origin		HII regions	HII regions	HII regions	PDR	Giant molecular clouds	Diffuse ISM
Absorption lines		Hydrogen, metals	Hydrogen, metals	Hydrogen			HI(21 cm)
Origin		Stellar atmosphere, ISM, IGM	Stellar atmosphere	Stellar atmosphere			Diffuse ISM

Notes: PAH: polycyclic aromatic hydrocarbons; PDR: photodissociation region; ISM: interstellar medium; IGM: intergalactic medium.

References

Astrophysics, decoding the cosmos, Irwin, Wiley
Astrophysics in a nutshell, Maoz, Princeton University Press
A panchromatic view of galaxies, Boselli, Wiley
Galaxies in the Universe: an introduction, Sparke and Gallagher, Cambridge University Press

Quiz to test your knowledge: *if you cannot answer to these questions you must go back to your course before coming to the tutoring lecture related to this chapter.*
WARNING: answering to these questions does not mean that you know all you need to know about the chapter….

1. Which relation links the luminosity and the flux of an astronomical source?
2. How do you define the angular resolution of an observation?
3. Give the definition of the magnitude relative and absolute and the relation linking them. What is the AB system?
4. Characterize the emission of a dense medium and of a diffuse medium

Chapter 2

Astronomical distances

Astronomers observe a two dimensional universe and must have an access to the third dimension. It has been one of the most difficult and challenging task of astronomers to measure robust distances for stars and external galaxies.

The distance scale starts from nearby stars for which direct measurements of distance have been performed for a long time (since the antiquity) and extends up to cosmological distances and the most distant galaxies ever known. Extragalactic distances are only measured with indirect methods. This succession of methods is also called the cosmic distance ladder

We will begin by briefly describing the methods to measure stellar distances before explaining how to access to the distance of galaxies

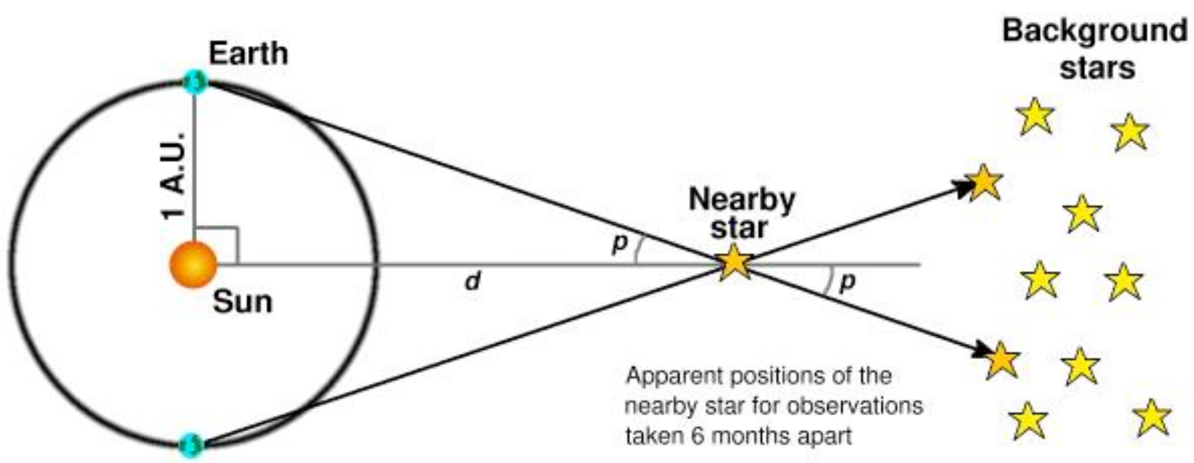
A. Stellar distances

1. Trigonometric parallax

When the earth orbits the Sun, the nearest stars seem to move relative to the more distant ones which appear fixed (their apparent motion is negligible).

This method is called trigonometric parallax

https://blogs.stsci.edu/livio/2014/09/30/an-astronomers-guide-to-the-galaxy/



Let us note a the Sun-Earth distance ($a=1$ AU$=149.6\ 10^9$ m) and d the distance of the nearby star:

$$\tan p \approx p \text{ (radians)} = a/d$$

Astronomers decide to measure angles in arcsec and distance in parsec. One parsec (1 pc) is the distance corresponding to a parallax of 1 arcsec i.e the distance to which the Sun-Earth distance corresponds to 1 arcsec

$$p \text{ (arcsec)} = a/d = 4.85\ 10^{-6} \text{ rad}$$

$$d = 1 \text{ pc} = 3.0856\ 10^{16} \text{ m}$$

Note: the astronomers do not use the light year (1pc corresponds to 3.26 light years

The first trigonometric parallax was measured in 1838 by Bessel, it was for the star 61 Cygni and p= 0.3 arcsec. The nearest star, Alpha Centauri has a parallax of 0.76 arcsec (and a distance of 1.3 pc).

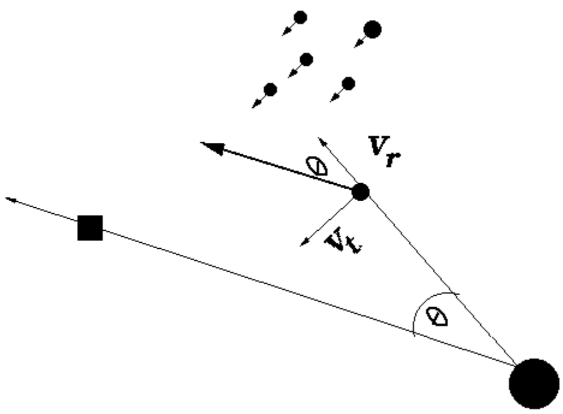
Trigonometric parallaxes (astrometric science) is difficult and time consuming. In the 80's a satellite, Hipparcos, achieved a precision of 0.002 arcsec for more than 100 000 stars. The GAIA mission is now surveying one billion stars with a precision of 20 (brightest stars) to 200 μarcsec (faintest stars).

The Hubble Space Telescope also achieved measurements with a similar precision but on a limited number of targets.

2. Other geometric methods and parallaxes

Proper stellar motions are used to reach larger distances (prior to the GAIA mission). Groups or cluster of stars gravitationaly bounded exhibit a global motion (peculiar motions of individuals average to 0). The observations must be conducted many years apart, so that the small displacements accumulate. This method is called statistical parallax or convergent point method. The distance obtained for the Hyades with this technics was an important step in the distance ladder.

The stars (black points) of the cluster seem to move to the convergent point (filled square). The angle between the convergent point and the star is measured, θ. The tangent velocity μ is measured over large periods of time in arcsec yr^{-1}. The radial velocity is measured by Doppler shift m s^{-1} and if v is the true velocity then and v_t the tangent velocity in ms^{-1} :



$v_t = v \sin\theta$

$v_t = 4.74\ 10^3\ \mu / p$, p is the parallax of the star

(1AU per year is equivalent to 4.74 10^3 m s^{-1})

and with $v_r = v \cos\theta$

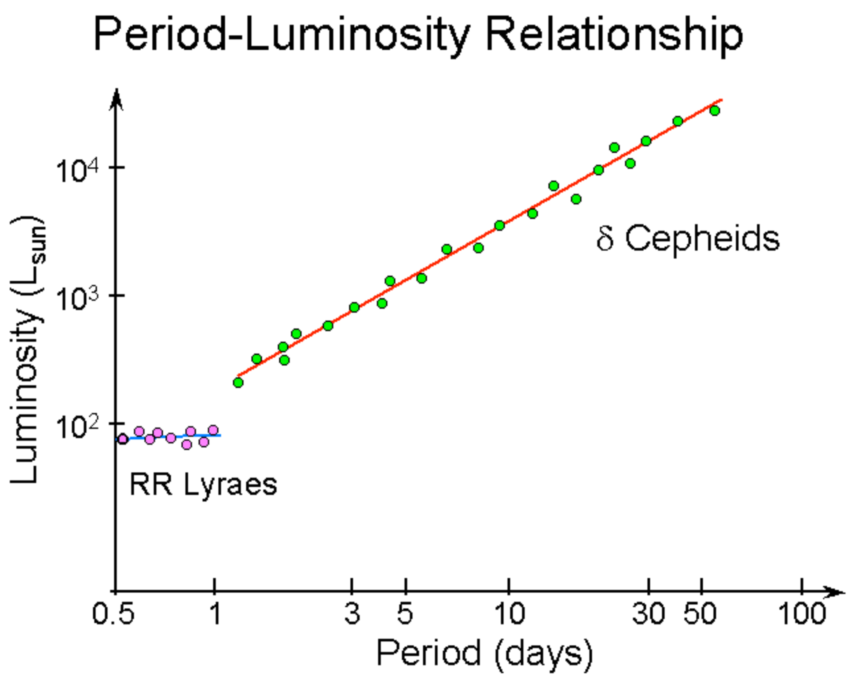
$p = 4.74\ 10^3\ \mu / v_r \tan\theta$

3. Using stellar candles

Some stars have very specific properties which help to identify them, and universal luminosities

a. RR Lyrae; they are variable stars, their luminosity varies with a period of few days. These stars are evolved, old stars are found in the oldest cluster of the Galaxy. Their absolute magnitude is the same for all these stars (M_V=0.5 mag) but they are intrinsically not very luminous and can only be detected at a distance lower than 1 Mpc, from ground-based telescopes

b. Cepheid stars: these variable stars show a relationship between their intrinsic luminosity and their period of variation. This relation was first defined in the Large Magellanic Cloud. They are intrinsically luminous stars (M_v~-7 mag) and can be detected out of a distance of ~15 Mpc with the Hubble Space Telescope, giving access to the galaxies close to the Milky Way. Cepheids are found in young, open cluster, which leads to difficulties to calibrate their luminosity-period relation. The periods of Cepheids can be as large as 50 days.



http://www.astronomy.ohio-state.edu/~pogge/

Note that all these stars are much more luminous than the Sun

273 Cepheids have measured parallaxes with Hipparcos, it gives the zero-point calibration of the period-luminosity relation and GAIA will observe many more such stars. However the calibration remains difficult because of the uncertainties about the properties of the stars and of their surrounding medium. The Cepheids are of fundamental importance to measure distances of nearby galaxies. The first great discovery of Erwin Hubble was from his observation of Cepheids in Andromeda: he found that they were much farther away than the ones in the Milky Way or the Large Magellanic Cloud and concluded that Andromeda is a separate galaxy.

4. Other methods

A number of other methods, based on the properties of some stars are used to give stellar distances. We can mention Novae, Supernovae, red giants or supergiants

The spectroscopic parallax: we will see that a star can be characterized from its spectrum. Its absolute magnitude can be fixed by its spectral type, and the measure of the apparent magnitude gives the distance (cf chapter 2).

The method is more accurate if a collection of stars is used. Since the spectral type of a star depends on its age, cluster of stars on same age are used. The main sequence (a sequence in a colour magnitude diagram where most of the stars lie) is then fitted and the average distance of the cluster measured.

We will go back to this method in chapter 3.

B. Distance to galaxies

1. Doppler effect and redshift

When a source emitting a wave is moving with respect to the observer, the wavelength of the emitted wave is changed when measured by the observer. It is a classical process first observed by Doppler and then explained by Fizeau. This Doppler effect is of fundamental importance in astronomy, since astronomical sources are globally receding from each other.

Let us consider a source of light S receding at speed v from the observer O. This source emits a wave of wavelength λ_e. T_e is the period of the wave. S emits a wavelength between t_1 and t_2 with $T_0 = t_2 - t_1$. What is the wavelength of the wave received by O?

The signal emitted at t_1 arrives at t_1' and travels r_1, the signal emitted at t_2 arrives at t_2' and travels r_2, and the period of the received wave is

$T' = t_2' - t_1'$

$r_1 = (t_1' - t_1)/c$ and $r_2 = (t_2' - t_2)/c$, c is the speed of light

$T' = (t_2' - t_2) - (t_1' - t_1) + (t_2 - t_1)$ and

$T' = [(r_2 - r_1)/c] + T$

At first order $r_2 - r_1 = v_r \, . \, T$ and $T' = (1 + v_r/c) \, . \, T$

Going back to the wavelengths emitted (λ_e) and observed (λ_0) : $\lambda_e = c\,T$ and $\lambda_0 = c\,T'$

$\lambda_0/\lambda_e = (1 + v_r/c)$

$\Delta\lambda = \lambda_0 - \lambda_e$ and $\boxed{\Delta\lambda/\lambda_e = v_r/c}$

This demonstration is in the non relativistic case (low speeds) and is not valid for high speeds. It applies to stellar movements. In most cases the velocities are low enough to apply the classical, non relativistic Doppler shift.

Relativistic particles can be found in some objects and in this cases the calculation in the framework of special relativity must be performed.

Neither the classical mechanics nor the special relativity is appropriate to our universe which is in an accelerating expansion. In this case the general relativity applies. Practically on line simulators give the exact calculation in this case, with the good set of cosmological parameters (e.g. http://ned.ipac.caltech.edu/help/cosmology_calc.html) $v_r > 0$ for an outward direction and $v_r<0$ for an inward direction, it leads to an increase of the wavelength for a source receding and a decrease of the wavelength for a source approaching.

The redshift is defined as $\boxed{z = \Delta\lambda/\lambda_e}$

"redshift" is for the shift towards red of lines observed in the spectra of galaxies which are receding.
The radial speed is then $v_r = cz$

1. Hubble's law

In 1929 Erwin Hubble found a relationship between the speed of recession of galaxies (measured from the Doppler shift) and the distance of these galaxies (the few ones whose distances were known).

$$\mathbf{V_r = H_0\, d}$$

This relation only holds for nearby galaxies; practically for redshifts lower than 0.8
The value of the Hubble constant varies with time, the subscript 0 is for the current time.
It is measured accurately, after big debates during the whole twentieth century.
The value obtained by the Hubble Space Telescope with cepheids: $H_0 = 72$ +/- 8 km s^{-1} Mpc^{-1}
The missions aimed at measuring the cosmological parameters (WMAP and PLANCK) found respectively:

$H_0 = 71.9$ +2.6/- 2.7 km s^{-1} Mpc^{-1} for WMAP and

$H_0 = 67.8$ +/- 1.4 km s^{-1} Mpc^{-1} for PLANCK

In the example below, it can be seen that the galaxies are not exactly on the Hubble's law, it is because they have peculiar velocities which are added to the cosmological Hubble law. The bunch of galaxies at a similar distance and with different velocities are from the Virgo cluster, they lie all at a same distance from us, and their velocity is perturbed by gravitational effects with their neighbours.

The measure of the redshift is used to infer distances of galaxies. The accuracy of the method relies on the calibration of the Hubble's law.

The Hubble law is explained by the cosmic expansion since the Big Bang. A very crude estimate of the age of the universe can be inferred from the Hubble's law.

For a constant speed the distance time relationship is

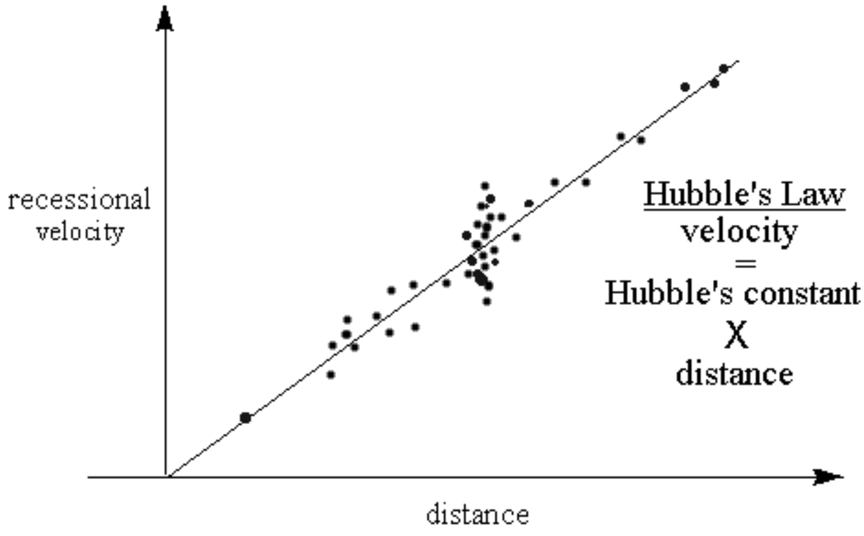
D = V t and t = D/V and if we compare with the Hubble's law

$1/H_0 = t$, t is the time since the expansion has started, it is called the Hubble time.

With $H_0 = 72$ km s^{-1} Mpc^{-1}

$t = 1/72 .(10^6).(3.0856\ 10^{13})$

$= 4.3\ 10^{17}s = 13.6\ 10^9$ yr, which is a good approximation for the age of the universe



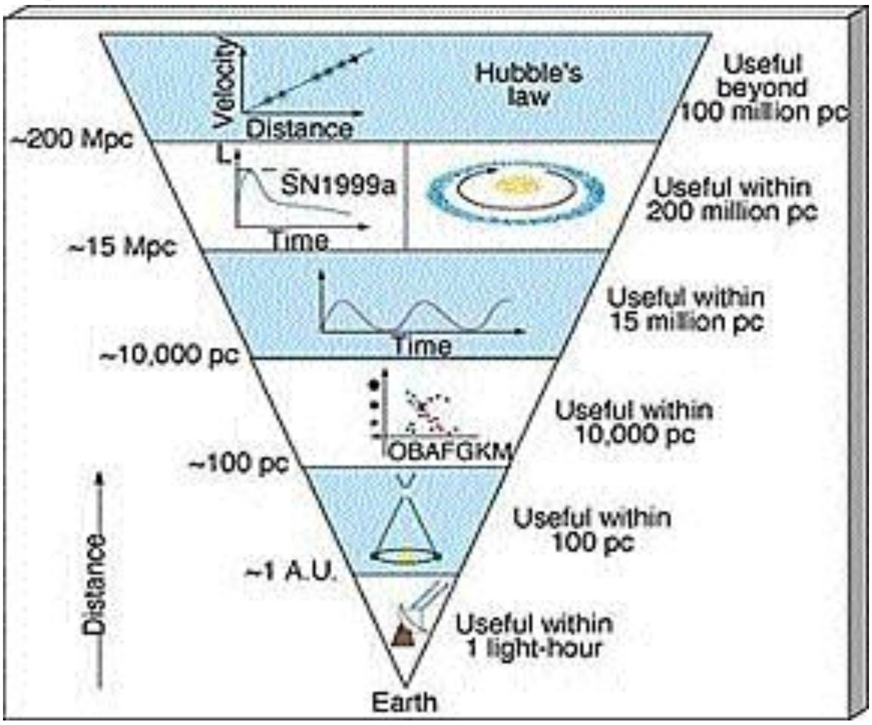
http://www.astro.cornell.edu/academics/courses/astro201/hubbles_law.htm

The value of the Hubble constant remains debated, and more importantly varied during the last decades. Therefore astronomers often prefer to give their results in a form which allows to change the value of H_0.
They define h as $H_0 = 100$ h km s^{-1} Mpc^{-1}

2. The cosmic distance ladder

Different estimators are used to measure distance, they are not independent since most of them (if we exclude the measure of the cosmological parameters) rely on the measures performed on nearby stars (parallaxes) or cepheids.

A very simplified illustration of this "ladder" is given below together with typical distances
From http://staff.on.br/jlkm/astron2e/AT_MEDIA/CH24/CHAP24AT.HTM



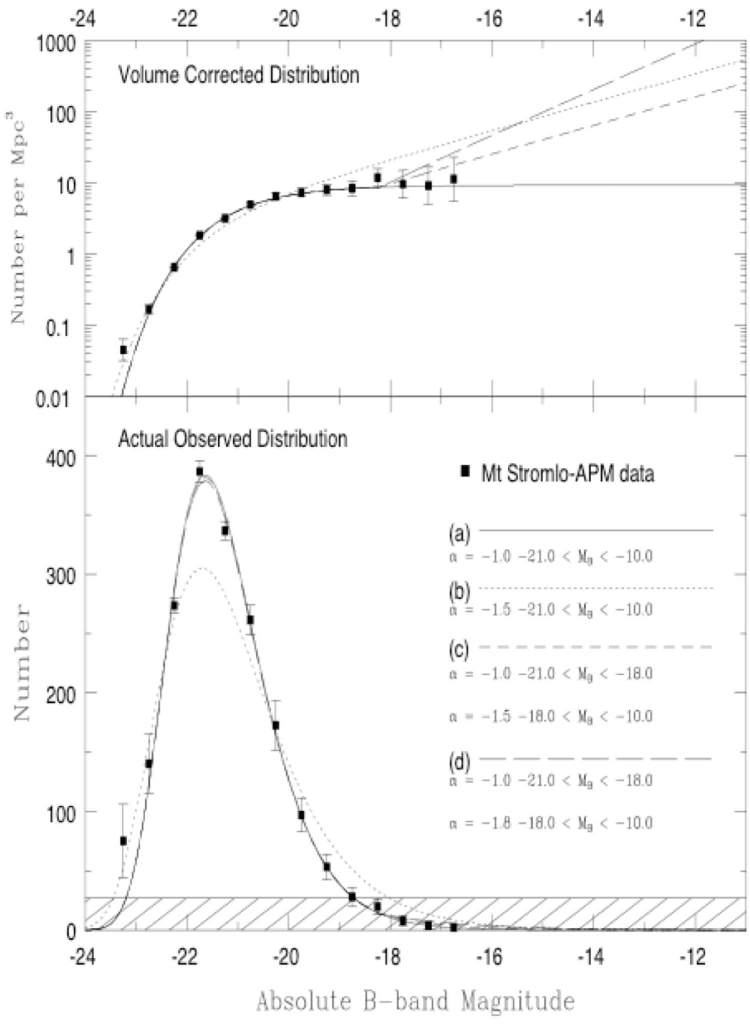
3. Observational biases

Astronomical sources of different luminosities are observed at different distances. Intrinsically bright objects will be detected preferentially. It is a well known selection bias, called the Malmquist bias, which affects flux limited surveys of stars or galaxies.

Objects which are intrinsically more luminous are observed at a greater distance than fainter sources, creating false values of densities. It also affect the average absolute magnitude and average distances induced for a sample of objects . This effect has led to many spurious claims in the field of astronomy. It is important to correct for it.

1. **Limiting the sample**: the most obvious way to correct for the Malmquist bias is to use only a fraction of the dataset in which all the objects studied can be detected. Practically one takes the lowest luminosity (the largest absolute magnitude) and cut the sample to distances where these objects are observed. Unfortunately, most of the time such a method wastes a lot of good data

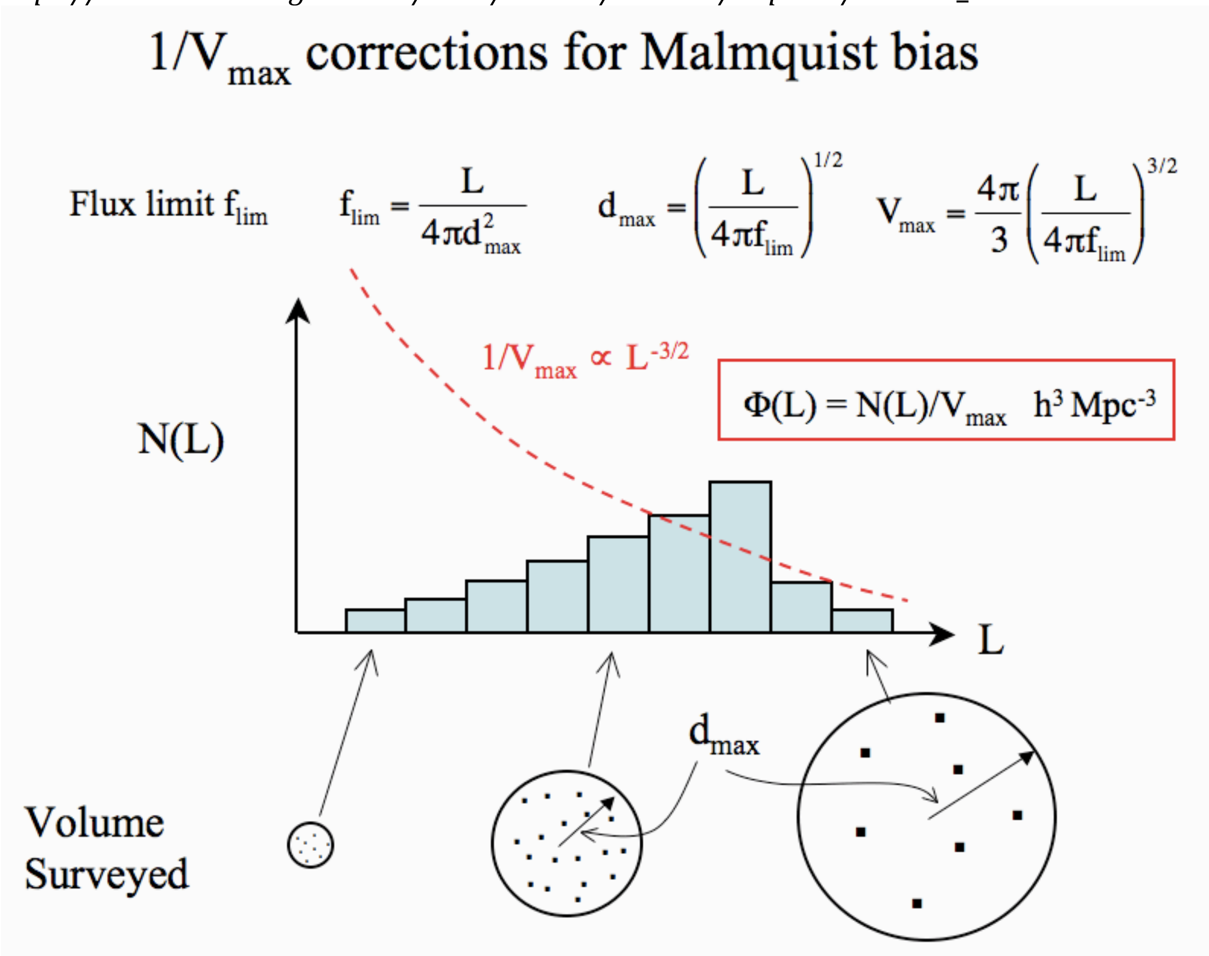
2. **Traditional correction**: Malmquist showed that for a Gaussian distribution of absolute magnitude (M_0, σ), the mean magnitude of the sample is:
$M = M_0 - 1.38\ \sigma^2$, this relation will be established in the exercises.
This correction applies under several assumptions:
-there is no extinction which affects the light
-the luminosity distribution does not depend on distance and is approximated by a Gaussian function (M_0, σ)
-the distribution of the sources is uniform



3. **Volume weighting:** it is very commonly used in extragalactic surveys. Each source contributes to the absolute magnitude (or luminosity) distribution with a weight corresponding to the inverse of the volume in which is could lie and be bright enough to be observed. It is called the V_{max} method, each source being defined with the maximum volume where it can be observed.
This maximum volume is estimated as a sphere with a radius D_{max} given by the distance modulus, using the object's absolute magnitude and the limiting apparent magnitude of the flux limited survey.
$V_{max} = \Omega/3\ D_{max}^3$ where Ω is the size of the survey area in steradian.
The plots (from *Driver & Philipps 1996 ApJ 469, 529)* represent the observed distribution of B magnitude of galaxies (lower panel), and the distribution corrected by the V_{max} method (upper panel

The next illustration explains the calculation of V_{max} with fluxes as well as the corrections to apply to obtain the corrected function shown above

https://www.astro.virginia.edu/class/whittle/astr553/Topic04/Lecture_4.html



Quiz to test your knowledge: *if you cannot answer to these questions you must go back to your course before coming to the tutoring lecture related to this chapter.*
WARNING: answering to these questions does not mean that you know all you need to know about the chapter….

1. What is the redshift of a source?
2. Define the Hubble's law
3. Enumerate the main steps of the distance ladder
4. What is the volume weighting of an observable sample

Chapter 3

Stellar radiation: basic properties of stars and classification

In this chapter we study the basic observed properties of stars and the main relations between these properties. This knowledge is the basis of all branches of Astrophysics which involve stars: interstellar medium, galaxies, nearby and distant, first stars, stellar clusters etc…

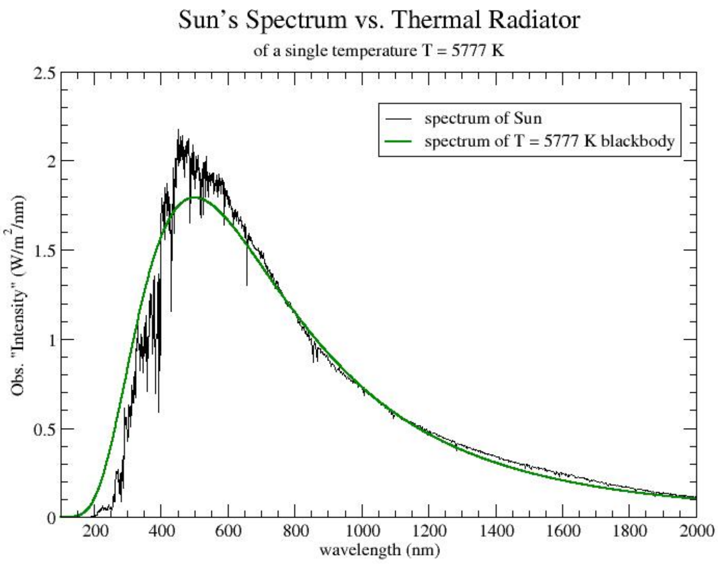
I. Stellar spectra

A star is a ball of hot gas held together by gravity. In the central part of the star, exothermic nuclear reactions take place that mainly convert hydrogen into helium.

This radiation interacts with the surrounding matter, up to the surface of the star. The mean free path of any photon is very long and we only observe the light emitted by the external part of the star.

As a first approximation stars radiate as black bodies because of this very long free path for photons produced in the center of the star. Actually, the spectral emission of a star differs from the one of a black body, by the presence of absorption lines and breaks, as can be seen below on the spectral emission of the Sun compared to the spectrum of a black body.

http://homepages.wmich.edu/~korista/phys106.html



A thermal equilibrium between particles and photons take place inside the gas which is described by :

- The Planck law (Black body spectrum) for the photons
- The laws of Maxwell, Boltzmann and Saha for the particles

II Laws of the thermal equilibrium, main properties of stars

A. Black Body (BB) radiation

1. The Planck function

The energy density per frequency interval is:

$$u_\nu = \frac{8\pi h \nu^3}{c^3} \frac{1}{e^{(h\nu/kT)} - 1}$$

h is the Planck constant, k the Boltzmann constant, T the temperature in Kelvin, c the speed of light

u_ν is given in J m^{-3} Hz^{-1} , it is an isotropic emission, and u_ν is the emission integrated over all directions.

The specific intensity of the Black Body (BB) is defined as the energy emitted, passing through a unit area (and perpendicular to it) per unit time:

B_ν is expressed per unit of frequency and solid angle (sr) in W m^{-2} sr^{-1} Hz^{-1}

The BB radiation is isotropic: the energy in a given direction is

$\frac{du_\nu}{d\Omega} = \frac{u_\nu}{4\pi}$, $d\Omega$ is the solid angle element

To obtain the specific intensity we multiply this energy with the velocity to obtain a flux per unit area per unit time (it is the energy going through the unit area per unit time)

And $B_\nu = c\frac{du_\nu}{d\Omega} = c\frac{u_\nu}{4\pi} = \frac{2h\nu^3}{c^2}\frac{1}{e^{(h\nu/kT)} - 1}$

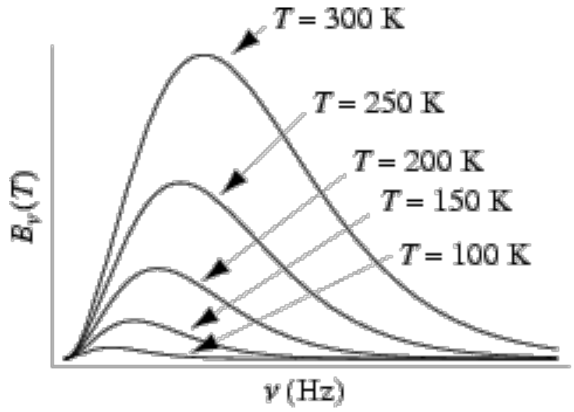
Rather than considering intensity per photon frequency interval we can also define an intensity (or any other energetic quantity) per photon wavelength interval.

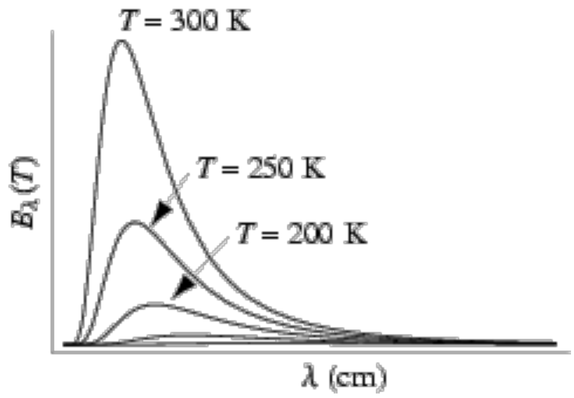
To go from B_ν to B_λ we must recall that the energy in a given interval, defined either in wavelength or in frequency, must be the same:

$B_\nu\, d\nu = B_\lambda\, d\lambda$ and $B_\lambda = B_\nu\, d\nu/d\lambda = B_\nu\, c/\lambda^2$

$$B_\lambda = \frac{2hc^2}{\lambda^5}\frac{1}{e^{(hc/\lambda kT)} - 1}$$

expressed in W m^{-2} sr^{-1} nm^{-1} (the wavelength can be expressed in nm, μm, cm, m and even in Å)





2. Peak of the Planck function

The peak of the Black Body is given by taking $dB_\nu/d\nu = 0$ or $dB_\lambda/d\lambda = 0$

Which leads to the two formulae of the Wien's law:

$\lambda_{max}T = 0.29$ cm K

and $h\nu_{max} = 2.8\ k\ T$

note that $\nu_{max} \neq c/\lambda_{max}$

3. Approximations of the Planck function

Far from its peak the Planck function is approximated with two simpler forms, very useful for applications:

- At low frequencies $h\nu << k\ T$
  $$B_\nu = \frac{2h\nu^3}{c^2}\frac{1}{e^{h\nu/kT}-1} \approx \frac{2h\nu^3}{c^2}\frac{1}{h\nu/kT} \approx \frac{2k\nu^2}{c^2}T$$
  It is the Rayleigh-Jeans approximation for large wavelength, mainly used for radio measurements
- At high frequencies $h\nu >> k\ T$ $B_\nu \sim e^{(-h\nu/kT)}$
  It is called the Wien tail of the distribution, useful in visible for most of the stars

4. Stefan-Boltzmann law

The Stefan-Boltzmann law relates the total energy or intensity of a Black Body to its temperature

$$B = \int_0^\infty \frac{2h\nu^3}{c^2}\frac{1}{\exp(h\nu/kT)-1}d\nu = \int_0^\infty \frac{2h\nu^3}{c^2}\frac{1}{\exp(x)-1}\frac{kT}{h}dx$$

$$B = \int_0^\infty \frac{2k^4T^4}{c^2h^3}\frac{x^3}{\exp(x)-1}dx = \frac{2k^4T^4}{c^2h^3} \times \frac{\pi^4}{15}$$

with $$\int_0^\infty \frac{x^3}{\exp(x)-1}dx = \frac{\pi^4}{15}$$

and $$\boxed{B = \int_0^\infty \frac{2h\nu^3}{c^2}\frac{1}{\exp(h\nu/kT)-1}d\nu = \sigma/\pi\ T^4}$$

with $\sigma = \pi^5/15\ .\ [2\ k^4/(c^2h^3)] = 5.67\ 10^{-8}\ W\ m^{-2}K^{-4}$

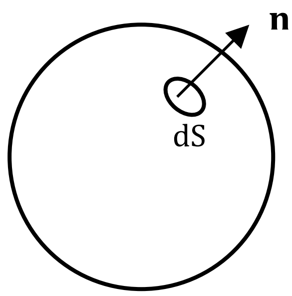
With a similar calculation:

$$u = \int_0^\infty \frac{8\pi h\nu^3}{c^3}\frac{1}{\exp(h\nu/kT)-1}d\nu = a\ T^4$$ with $a = 4\ \sigma/c$

5. Total power emitted by a star of radius R

B is the specific intensity integrated over all frequencies (as defined above).

Let us first consider the flow of energy emerging from dS on the outer surface. We multiply B by the cosine of the angle between the intensity and the vertical **n** and we integrate over the solid angle on the half sphere facing outward. Then we multiply by the total surface of the sphere



$d\Omega = 2\ \pi \sin\theta\ d\theta$

$$L = \frac{\sigma}{\pi} T^4 \times (4\pi R^2)\int_0^{\pi/2} \cos\theta\ 2\pi\ \sin\theta\ d\theta$$

and $\boxed{L = 4\ \pi\ R^2\ \sigma\ T^4}$

per unit frequency $L_\nu = 4\ \pi\ R^2\ \pi\ B_\nu$

if we consider the flux that an observer at a distance d from the star can measure:

$f_\nu(d) = \pi\ B_\nu\ R^2/d^2$

6. Diagram (L,T) of stars

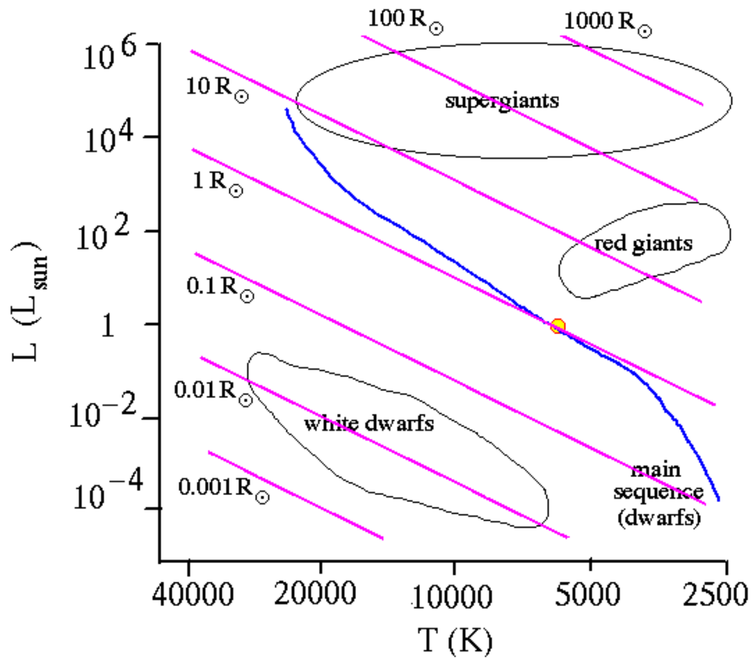
This diagram is also called the Hertzsprung-Russel diagram: the temperature of the black body corresponding to the same total luminosity as that of the star is plotted against the stellar luminosity. We will go back to this diagram later with details. Here we use it to illustrate the link between luminosity, size and temperature of stars.

An illustration of the HR diagram is shown below, stars are found in specific areas of this diagram. Note that temperature is growing to the left.

Most stars (80-90%) lie on a diagonal strip called "main sequence" (blue line). The Sun is on this main sequence (yellow point). There are also other concentrations of stars named "red giants", "supergiants" or "dwarfs".

The size of stars is normalized to the Sun's radius called R_{sun} ($R_{sun} \approx 7\ 10^8$ m). The relation $L = 4\pi R^2 \sigma T^4$ is plotted as pink lines for different radius. It appears that supergiants have luminosities and sizes order of magnitudes higher than the main sequence stars which are called "dwarfs" because of their moderate radius (the Sun is a dwarf).

http://abyss.uoregon.edu/~js/ast122/lectures/lec11.html



B. Distribution laws of particles

1. Maxwell distribution

At a given temperature (i.e. thermal equilibrium) the particle of a non relativistic gas achieve an equilibrium distribution of velocities known as the Maxwellian distribution:

The number of particles with velocities comprised between v and v+dv is:

$$f(v)dv = \frac{dN}{N} = \left(\frac{m}{2\pi kT}\right)^{3/2} \exp\left(-\frac{mv^2}{2kT}\right) 4\pi\, v^2 dv$$

$\int f(v)dv = 1$ (using the Stirling integral G_2)

The most probable speed is given by the peak of the distribution f(v):

Derivating f(v):

$2 v_p - 2 m v_p^3 /(2kT) = 0$ and $v_p = (2kT/m)^{1/2}$

The average quadratic velocity is given by

$< v^2 > = \int_0^\infty v^2 f(v)dv = 3kT/m$ and

½ m <v²> = 3/2 kT

Note: All these calculations are performed using Gaussian integrals http://en.wikipedia.org/wiki/Gaussian_integral

2. Boltzmann and Saha distributions

When thermal equilibrium prevails the number density of atoms in a given state is given by the Boltzmann equation:

$n_B/n_A = (g_B/g_A) \exp[-(E_B-E_A)/kT]$

n is the number density of atoms at the quantum level (A or B), g is the degeneracy of the quantum level (A or B) and E the energy of the level (A or B).

As the temperature increases more energy is available to ionize the atoms (through radiative or collisional processes) and the gas consists of neutral atoms, ions and free electrons.

The Saha equation describes the degree of ionization of the plasma as a function of the temperature (thus at thermal equilibrium) , density, and ionization energies of the atoms.

$$\frac{n_{i+1}}{n_i} n_e = 2\left(\frac{2\pi\, m_e kT}{h^2}\right)^{3/2} \frac{g_{i+1}}{g_i} \exp\left(-\frac{\chi_i}{kT}\right)$$

n_i is the number density of ions in the i-th ground state of ionization, g_i the degeneracy of the ground state of the ion, χ_i is the ionization potential from state i to state i+1

n_e is the number density of electrons, m_e the electronic mass

Note: Demonstration of the Saha equation (needs knowledge in statistical physics)

We use the Boltzman equation to derive the number ratio n_{i+1}/n_i .
The statistical weight of an ion in the lower ionization state i is g_i. The statistical weight of an ion in the upper ionization state i+1 is g_{i+1} multiplied by the number of possible states in which the free electron may be put.
In a cell of a phase space with a volume h^3 there are two possible states for an electron, corresponding to the possible orientations of its spin.
The energy of the free electron with a momentum p is $E = p^2/2m$. The number of cells available for free electrons with a momentum between p and p + dp is $v_e/h^3\, 4\pi p^2\, dp$, where v_e is the unit volume space available per electron.
Using the Boltzmann equation and integrating over all the possible energy for the free electron:

$$\frac{n_{i+1}}{n_i} = \frac{g_{i+1}}{g_i} \int_0^\infty \exp\left(-\frac{\chi_i + p^2/2m}{kT}\right) \times 2\frac{v_e}{h^3} 4\pi\, p^2 dp$$

$v_e = 1/n_e$, n_e being the number density of free electrons

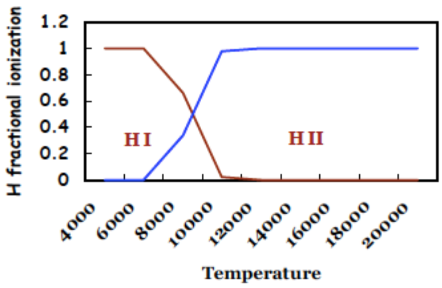
Setting $x = p^2/(2mkT)$

$$\frac{n_{i+1}}{n_i} = \frac{g_{i+1}}{g_i}\frac{1}{n_e}\frac{2}{h^3}(2mkT)^{3/2} e^{-\chi_i/kT} 2\pi \times \int_0^\infty \exp(-x)\, x^{1/2} dx$$

and $\int_0^\infty \exp(-x)\, x^{1/2} dx = \pi^{1/2}/2$ leads to the Saha equation

If the Boltzmann and Saha equations are applied to hydrogen atoms and ions (for a solar electronic density) we get

T (K)	n(H⁺) / n(H)	n(H) /[n(H) + n(H⁺)]	n(H⁺) /[n(H) + n(H⁺)]
4,000	2.46E-10	1.000	0.246E-9
6,000	3.50E-4	1.000	0.350E-3
8,000	5.15E-1	0.660	0.340
10,000	4.66E+1	0.021	0.979
12,000	1.02E+3	0.000978	0.999
14,000	9.82E+3	0.000102	1.000
16,000	5.61E+4	0.178E-4	1.000

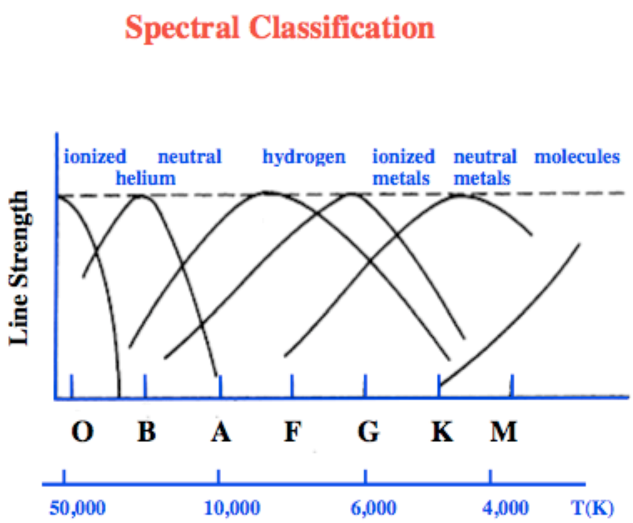


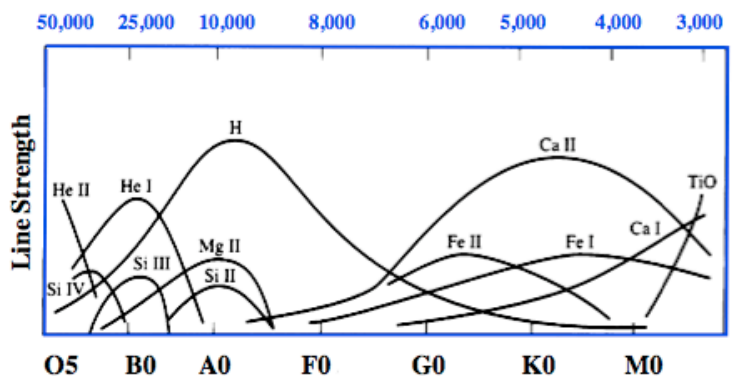
The hydrogen is fully ionized for temperatures higher than ~10 000 K

3. Spectral classification of stars

The application of the Boltzmann and Saha laws to hydrogen, helium and other atomic species and molecules leads to these plots. The presence of specific states (neutral, ionized at different levels) of any element depend on the temperature.

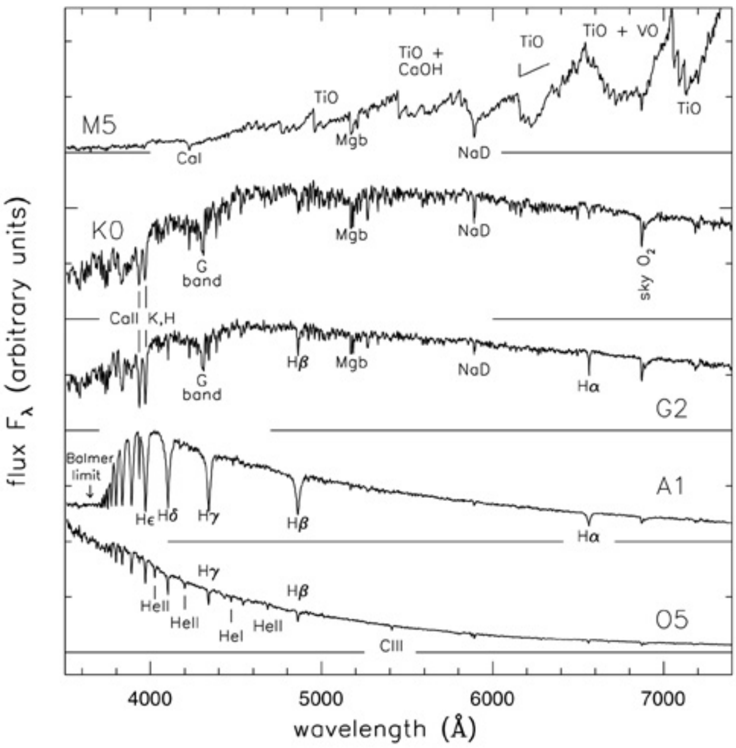
A spectral classification **O,B,A,F,G,K,M** from the hottest to the coldest stars is used.





Absorption lines of the corresponding species will be present in the spectra of stars and are also representative of the temperature of these stars.

Galaxies in the universe, Sparke & Gallagher, 2007



The list of the spectral classes with the temperature and most prominent lines is given here:

Spectral Class	Approximate Surface Temperature (K)	Noteworthy Absorption Lines	Familiar Examples
O	30,000	Ionized helium strong; multiply ionized heavy elements; hydrogen faint	Mintaka (O9)
B	20,000	Neutral helium moderate; singly ionized heavy elements; hydrogen moderate	Rigel (B8)
A	10,000	Neutral helium very faint; singly ionized heavy elements; hydrogen strong	Vega (A0), Sirius (A1)
F	7000	Singly ionized heavy elements; neutral metals; hydrogen moderate	Canopus (F0)
G	6000	Singly ionized heavy elements; neutral metals; hydrogen relatively faint	Sun (G2), Alpha Centauri (G2)
K	4000	Singly ionized heavy elements; neutral metals strong; hydrogen faint	Arcturus (K2), Aldebaran (K5)
M	3000	Neutral atoms strong; molecules moderate; hydrogen very faint	Betelgeuse (M2), Barnard's Star (M5)

Note that subclasses are defined as numbers from … (B1, B2 etc), the Sun is a G2 star.

We have seen above that stars of similar temperatures can have very different luminosities and sizes. Luminosity classes are thus defined.

Quiz to test your knowledge: *if you cannot answer to these questions you must go back to your course before coming to the tutoring lecture related to this chapter.*
WARNING: answering to these questions does not mean that you know all you need to know about the chapter….

1. Recall the main properties of a Black Body and the definition of the energy density and the specific intensity
2. What is the Wien's law? Which temperature can be estimated using this law?
3. Recall the Stefan Boltzmann law and the relation linking the luminosity, temperature and radius of a star assumed to radiate as a Black Body.
4. What is the HR diagram? Define the main sequence of stars
5. Give the main characterisics of the Spectral Classification of stars